Regular BPS black holes: macroscopic and microscopic description of the generating solution

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Abstract

In this paper we construct the BPS black hole generating solution of toroidally compactified string (and M) theory, giving for it *both* the macroscopic and microscopic description. Choosing a proper U–duality gauge the latter will be given by a bound state made solely of D–branes. The axionic nature of the supergravity solution will be directly related to non–trivial angles between the constituent D–branes (type IIB configuration) or, in a T–dual gauge, to the presence of magnetic flux on constituent D–brane world volumes (type IIA configuration). As expected, the four dimensional axion fields arise from the dimensional reduction of non–diagonal metric tensor components or Kalb-Ramond B field components for type IIB or type IIA cases, respectively. Thanks to this result it is then now possible to fill the *full* 56–dimensional U–duality orbit of $N = 8$ BPS black holes and to have a macroscopic and microscopic description of all of them.

Contents

1 Introduction and summary of the results

In the last 4 years the study of black hole solutions in supergravity and string theory has acquired a renewed interest. This is due, essentially, to some successful microscopic entropy countings which have given, for the first time [1, 2, 3, 4], a statistical interpretation of the Beckestein–Hawking entropy formula in the context of a consistent quantum theory of gravity, such as string theory. The general properties of both supersymmetric and near supersymmetric black holes have been systematically studied and many new results have been obtained.

Despite this progress, it is still not possible nowadays to have a complete control on the common microscopic properties underlying these black holes. Many efforts have been made in order to find a way of making these microscopic entropy countings more based on first principles rather than on case by case computations (see for example [5, 6, 7, 8, 9, 10, 11, 12]) and to understand which are the actual microscopic degrees of freedom underlying the microscopic/macroscopic matching. However, a definite answer has not been found, yet. In this respect, it would be useful to find a precise description, *both* at macroscopic and microscopic level, of the so–called generating solution of regular BPS black holes obtained in the framework of superstring (or M) theory compactified on the torus T^6 (T^7).

There are at least three known macroscopic generating solutions in the literature, [13, 14, 15]. However, their structure is too involved to allow a direct construction of the corresponding microscopic configuration. On the other hand, it was proposed in [16] one possible stringy (i.e. microscopic) structure for the generating solution but it was not possible to find its field theory description as a solution of the relevant supergravity theory.

In this paper we fill the above gap and finally find the $N = 8$ BPS black holes generating solution, giving for it *both* a macroscopic and a microscopic description in a clear and simple way. As expected, the solution will depend on five independent parameters which will have a precise physical meaning both at macroscopic and microscopic level. Despite the number of independent

parameters is five, the number of independent harmonic functions entering the solution will be four. This is somewhat expected and is related to the well known fact that within the five invariants of the U–duality group of toroidally compactified M–theory, i.e. $E_{7(7)}$, four are moduli–dependent and one is moduli–independent (and proportional to the entropy). Another feature of the generating solution is to have non–trivial axion fields switched on in it (this is related, as we shall see, to the intrinsic property of the central charge eigenvalues evaluated on the solution of being intrinsically complex, i.e. not all imaginary). On the microscopic side this will correspond to have a configuration of D–branes intersecting at non–trivial angles (IIB) or, in a T–dual picture, a D–brane configuration with non–trivial magnetic fluxes on the D–brane world–volumes (IIA). As expected, the non–trivial four dimensional axions will come from off–diagonal components of the ten dimensional metric G_{MN} in the type IIB case and of the antisymmetric tensor B_{MN} in the type IIA case, along the world volume of the D–branes (and constant with respect to the world volume coordinates). Switching off the magnetic flux one would recover a four parameter solution of D–branes orthogonally intersecting, which turns out to be a pure dilatonic one.

Once a macroscopic solution it is given, its specific microscopic counterpart it is not uniquely defined: it depends on how the solution is embedded in the original ten dimensional theory. Our choice is to use two suitable R–R embeddings recently defined in [17], in order to have, as anticipated, a stringy configuration made solely of D–branes. Relying on the structure of the superalgebra central charges and thanks to the geometrical control of the embedding of the solution in the full type II string theory, it will be quite easy to figure out the microscopic systems corresponding to our solution. Moreover, using the tools developed in previous papers ([15, 18, 19] and especially [17]) we are able to generate, from the above simple pure D–brane description, any other configuration, even pure NS–NS ones, i.e. those made of solely NS states (as fundamental strings, NS5–branes and KK–states)[2]. The property for the entropy of being an U–duality invariant ensures that all these U–dual configurations share the same entropy. And, as already stressed, the possibility of having a control both at macroscopic and microscopic level of all these configurations, could give some help in unraveling the very conceptual basis of the microscopic entropy counting.

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[2]As shown in [20], the generating solution of heterotic black holes is also a generating solution for the type II ones. It is the U–duality group which changes in the two cases and which specifies the U–duality properties of the solution. In the heterotic case is $U = SL(2) \times SO(6, 22)$ while in the present case, i.e. type IIA, type IIB or M–theory compactified on tori (T^6 or T^7, respectively), we have $U = E_{7(7)}$.

2 The macroscopic generating solution

Let us start with the macroscopic description of the generating solution. It has been shown in [18] that the BPS black holes generating solution of toroidally compactified type II string is also solution of a consistent truncation of the relevant $N = 8$ supergravity effective theory, the so called STU model. This is a $N = 2$ effective model characterized by the graviton multiplet and just three vector multiplets, each of them containing two scalar fields[3]. Therefore its full bosonic field content is: a graviton, four vector fields (i.e. eight charges, (p_Λ, q_Λ) where $\Lambda = 0, 1, 2, 3$) and three complex scalars $z^i = a_i + i b_i$ spanning the Special Kähler manifold $\mathcal{M}_{STU} = [SL(2, \mathbb{R})/SO(2)]^3$ (b_i being the dilatonic fields and a_i the axions). We are not going to describe the detailed structure of the model since it has been described in a complete way in [15, 19]. In fact, we will use the same conventions and notations adopted in those papers. Let us then just briefly summarize the procedure to follow in order to derive the generating solution in the framework of the STU model, leaving to the next section the discussion of its embedding in the $N = 8$ theory. The BPS condition is equivalent to imposing the vanishing of the fermion supersymmetry variation along the *Killing spinor* ξ_a direction:

$$\begin{aligned} \delta_\xi \text{fermions} &= 0 \\ \gamma^0 \xi_a &= \pm \frac{Z}{|Z|} \epsilon_{ab} \xi^b \quad \text{if} \quad a, b = 1, 2 \end{aligned} \tag{2.1}$$

$Z(z, \overline{z}, p, q)$ being the $N = 2$ supersymmetry central charge. We adopt the following ansätze for the metric, Killing spinor, scalars and vector field–strengths:

$$\begin{aligned} ds^2 &= e^{2\mathcal{U}(r)} dt^2 - e^{-2\mathcal{U}(r)} d\vec{x}^2 \qquad \left(r^2 = \vec{x}^2 \right) \\ \xi_a(x) &= f(r) \epsilon_a \\ z^i(x) &= z^i(r) \\ F^\Lambda(r) &= \frac{p_\Lambda}{2r^3} \epsilon_{krs} x^k dx^r \wedge x^s - \frac{l_\Lambda(r)}{r^3} e^{2\mathcal{U}(r)} dt \wedge \vec{x} \cdot d\vec{x} \quad , \quad \Lambda = 0, 1, 2, 3 \end{aligned} \tag{2.2}$$

l_Λ being the moduli–dependent electric charges defined in [19]. From the BPS conditions (2.1) we may derive an equivalent system of first order equations for the scalars and metric function $\mathcal{U}$:

$$\begin{aligned} \frac{dz^i}{dr} &= \mp 2 \left(\frac{e^{\mathcal{U}(r)}}{r^2} \right) h^{ij^\star} \partial_{j^\star} |Z(z, \overline{z}, p, q)| \\ \frac{d\mathcal{U}}{dr} &= \mp \left(\frac{e^{\mathcal{U}(r)}}{r^2} \right) |Z(z, \overline{z}, p, q)| \end{aligned} \tag{2.3}$$

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[3] BPS solutions of $N = 2$ supergravity have been studied, for instance, in [21, 22, 23, 24, 25, 26].

The explicit expression of the right hand side of eq.s (2.3) is quite involved and may be derived from the equations in [19] (computed using the too restrictive condition $Z = -\overline{Z}$) by multiplying their right hand side by $\sqrt{Z/\overline{Z}}$ or by $\sqrt{\overline{Z}/Z}$ as far as the equations for the scalars or the one for $\mathcal{U}$ are concerned, respectively.

In order to characterize the generating solution we need to compute the skew–eigenvalues $\{Z_\alpha\} = \{Z_{\hat{i}}, Z_4\}$ ($\alpha = 1, \ldots, 4$) of the $N = 8$ central charge Z_{AB} in terms of the supersymmetry and *matter* central charges $\{Z, Z_i\}$ of the $N = 2$ model, using the following relations:

$$Z = i\,Z_4 \quad , \quad Z^i = h^{ij^\star}\nabla_{j^\star}\overline{Z} = \mathbb{P}^i_{\hat{i}} Z^{\hat{i}} \quad (i, \hat{i} = 1, 2, 3) \tag{2.4}$$

where $\mathbb{P}^i_{\hat{i}} = 2b_i(r)$ is the vielbein transforming the rigid indices $\hat{i}$ (the one characterizing the eigenvalues of the $N = 8$ central charge in its normal form) to the curved indices i of the STU scalar manifold (see [19], section 3, for details). Following [19], the explicit expression of the $N = 8$ central charge eigenvalues in terms of quantized charges and moduli is given in the appendix. The five U–duality invariants characterizing a generic solution are the four norms $|Z_\alpha|$ and the overall phase $\Phi = \sum_\alpha \mathrm{Arg}(Z_\alpha)$, [27]. These invariants may be combined to give four moduli–dependent invariants and one moduli–independent invariant (the quartic invariant of the U–duality group). The generating solution is defined [15] as the BPS black hole solution depending on the least number of parameters such that, on the point of the moduli space ϕ^∞ defining the boundary condition at radial infinity of its scalar fields, the five invariants can assume all 5–plets of values (consistent with the positivity condition of the quartic invariant). A necessary condition for the generating solution is thus to depend only on five quantized charges, obtained from the original 8 by suitably fixing the $SO(2)^3$ gauge. Our choice for the gauge fixing is $p_0 = q_2 = q_3 = 0$. As an a–posteriori check that the solution is a generating one, it is necessary to verify that the five invariants computed in the corresponding ϕ^∞ are independent functions of the five remaining charges: q_0, q_1, p_1, p_2, p_3.

With the above gauge choice the system of first and second order differential equations symplifies considerably and the fixed values for the scalar fields (namely the values the scalars get at the horizon, [28]) turn out to be the following ones:

$$
\begin{aligned}
a_1^{fix} &= -\frac{q_1 p_1}{2p_2 p_3} \quad , \quad & b_1^{fix} &= -\sqrt{\frac{q_0 p_1}{p_2 p_3} - \frac{1}{4}\left(\frac{q_1 p_1}{p_2 p_3}\right)^2} \\
a_2^{fix} &= \frac{q_1}{2p_3} \quad , \quad & b_2^{fix} &= -\sqrt{\frac{q_0 p_2}{p_1 p_3} - \frac{1}{4}\left(\frac{q_1}{p_3}\right)^2} \\
a_3^{fix} &= \frac{q_1}{2p_2} \quad , \quad & b_3^{fix} &= -\sqrt{\frac{q_0 p_3}{p_1 p_2} - \frac{1}{4}\left(\frac{q_1}{p_2}\right)^2}
\end{aligned}
\tag{2.5}
$$

Let us now introduce the following harmonic functions:

$$H^i(r) = 1 + \frac{\sqrt{2}p_i}{r} \quad \text{with} \quad i = 1, 2, 3$$

$$H_0(r) = 1 + \frac{\sqrt{2}q_0}{r} \quad \text{and} \quad H_1(r) = g + \frac{\sqrt{2}q_1}{r} \quad, \quad g \equiv \frac{q_1}{p_1 + p_2} \tag{2.6}$$

where the above value for the parameter g is fixed by supersymmetry (first order equation for $\mathcal{U}$).

One can now see that the following ansätze for the $a_i(r)$, the $b_i(r)$ and the scalar function $\mathcal{U}(r)$:

$$a_1 = \frac{-H_1 H^1 + gH^2}{2H^2 H^3} \quad, \quad b_1 = -\sqrt{\frac{H_0 H^1}{H^2 H^3} - \frac{1}{4}\left(\frac{H_1 H^1 - gH^2}{H^2 H^3}\right)^2}$$

$$a_2 = \frac{H_1 H^1 - gH^2}{2H^1 H^3} \quad, \quad b_2 = -\sqrt{\frac{H_0 H^2}{H^1 H^3} - \frac{1}{4}\left(\frac{H_1 H^1 - gH^2}{H^1 H^3}\right)^2}$$

$$a_3 = \frac{H_1 H^1 + gH^2}{2H^1 H^2} \quad, \quad b_3 = -\sqrt{\frac{H_0 H^3}{H^1 H^2} - \frac{1}{4}\left(\frac{H_1 H^1 - gH^2}{H^1 H^2}\right)^2}$$

$$\mathcal{U} = -\frac{1}{4}\ln\left(H_0 H^1 H^2 H^3 - \frac{1}{4}(H_1 H^1 - gH^2)^2\right) \tag{2.7}$$

satisfies both the first and second order differential equations and hence is the solution we were looking for[4]. The values of the constants characterizing each harmonic function have been chosen in such a way to have 1) asymptotic flat space and 2) asymptotic unitary values for the dilatons b_i, so to have unitary radii of compactification. The corresponding point in the moduli space at infinity is thus:

$$\phi^\infty \equiv \begin{cases} a_1 = a_2 = 0;\ a_3 = g \\ b_i = -1 \end{cases} \tag{2.8}$$

Since the boundary values of the scalar fields at infinity define a bosonic vacuum of the theory, they characterize also the microscopic configuration realizing our solution in the opposite string coupling regime. In next section we shall describe two particularly simple microscopic configurations corresponding to the choice of ϕ^∞ in eq. (2.8).

Notice that the number of truly independent harmonic functions in eq.(2.7) is four, as expected, although the number of independent charges is five: q_0, q_1, p_1, p_2, p_3. Indeed, from the conditions (2.6), one sees that $H_1(r) = g\left(H^1(r) + H^2(r) - 1\right)$. Finally, according to the ansätze (2.2), the metric has the following form:

$$ds^2 = \left(H_0 H^1 H^2 H^3 - \frac{1}{4}(H_1 H^1 - gH^2)^2\right)^{-1/2} dt^2 - \left(H_0 H^1 H^2 H^3 - \frac{1}{4}(H_1 H^1 - gH^2)^2\right)^{1/2} d\vec{x}^2 \tag{2.9}$$

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[4]This result is consistent with the analysis in [26].

and the macroscopic entropy, according to Beckenstein–Hawking formula, reads:

$$S_{macro} = 2\pi \sqrt{q_0 p_1 p_2 p_3 - \frac{1}{4}(q_1 p_1)^2} \tag{2.10}$$

which is the expected expression for the entropy of a generating solution, [13, 14].

As anticipated, in order to check that the above solution is indeed a five parameters one, one has to work out the expression of the $N = 8$ central charge skew–eigenvalues on ϕ^∞. From the explicit expressions of the field dependent central charge in the appendix and from (2.8) it follows that:

$$
\begin{aligned}
Z_1(\phi^\infty, p, q) &= \frac{1}{2\sqrt{2}}\left[-2\frac{q_1 p_1}{p_1 + p_2} + i\left(q_0 + p_1 - p_2 - p_3\right)\right] \\
Z_2(\phi^\infty, p, q) &= \frac{1}{2\sqrt{2}}\left[2\frac{q_1 p_1}{p_1 + p_2} + i\left(q_0 - p_1 + p_2 - p_3\right)\right] \\
Z_3(\phi^\infty, p, q) &= \frac{1}{2\sqrt{2}}\left[0 + i\left(q_0 - p_1 - p_2 + p_3\right)\right] \\
Z_4(\phi^\infty, p, q) &= \frac{1}{2\sqrt{2}}\left[0 + i\left(q_0 + p_1 + p_2 + p_3\right)\right]
\end{aligned}
\tag{2.11}
$$

From the above equations it is clear that the 5 invariant quantities $|Z_\alpha(\phi^\infty, p, q)|$, $\Phi(\phi^\infty, p, q)$ are independent functions of the five charges q_0, q_1, p_1, p_2, p_3.

Switching off the fifth parameter q_1 our solution becomes exactly the four parameters one studied in [17] (indeed $H_1(r)|_{q_1=0} = 0$ and $g(q_1 = 0) = 0$). Indeed putting q_1 to zero the central charges Z_α become pure imaginary and the axion fields vanish uniformly.

3   The microscopic description of the generating solution

The microscopic counterpart of a four dimensional macroscopic solution is of course not uniquely defined. Indeed it depends on the interpretation of the four dimensional fields describing the supergravity solution in terms of dimensionally reduced ten dimensional ones. In the light of the analysis put forward in [29] and then completed in [17][5], the microscopic interpretation of our generating solution can be uniquely defined in terms of the embedding of the STU model within the original $N = 8$ theory (in particular of the embedding of the SLA generating the STU scalar manifold inside the SLA parametrized by the 70 scalars of the $N = 8$ theory). In [17], two main classes of embeddings of the STU model were defined (the embeddings within each class being related by S, T dualities): one in which the vector fields derive from NS–NS ten dimensional

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[5]Among the main goals of this analysis is the geometrical characterization, using solvable Lie algebra (SLA) techniques, of the scalar and vector fields in the $N = 8$ theory in terms of type IIA and IIB fields.

forms and the other in which the vector fields have a R–R origin (and the scalar fields are NS–NS, see in particular section 2 of that paper). In the latter class two representative embeddings were considered: if we denote by x^4, x^5, ..., x^9 the coordinates of T^6 and by x^0, x^1, x^2, x^3 the non–compact space–time coordinates, in one embedding, which was characterized from the type IIB point of view, all the three axions of the model come from metric tensor components (G_{45}, G_{67}, G_{89}), as opposite to the other, characterized from a type IIA point of view, in which all the axions come from B field components (B_{45}, B_{67}, B_{89}). In both cases the three dilatons are related to three combinations of the radii of the torus. The two embeddings are related by an operation of T–duality on the compact directions x^5, x^7, x^9. As we shall show in the sequel these two embeddings provide an interpretation of the generating solution (2.7) in terms of two T–dual microscopic configurations: a system of D3–branes at angles (type IIB embedding) and a system of D0 and D4–branes (type IIA embedding) with a magnetic flux in the world volume of the latter (giving therefore extra D2 and D0 charge, [30]). The magnetic flux (or, equivalently, the non–trivial angle in the dual type IIB configuration) will be the microscopic extra degree of freedom related to the fifth parameter q_1 characterizing the supergravity solution.

Analyzing the two embeddings from a SLA point of view, one can deduce, in the same way as it was done in [17] for the type IIA case (see in particular eq.s (3.14) and (3.15) of that paper), the subset of the weight basis of the **56** of $E_{7(7)}$ in terms of which the *magnetic* $y^n(\phi)$ $(n = 0, \ldots, 3)$ and *electric* $x_n(\phi)$ *dressed charges*[6] of the two *STU* model truncations are expressed. According to [17], the symplectic vector of dressed charges (y^n, x_n) is defined in the following way:

$$\begin{pmatrix} y^n(\phi) \\ x_n(\phi) \end{pmatrix} = -\mathbb{L}^{-1}(\phi) \begin{pmatrix} p^n \\ q_n \end{pmatrix} \tag{3.1}$$

where ϕ denotes a point in the scalar manifold and $\mathbb{L}(\phi)$ is the coset representative of the scalar manifold computed in the same point. As far as the type IIA embedding is concerned, a basis of weights for (y^n, x_n) was found in [17], and from table 3 of the same work the correspondent R–R vectors may be read off:

type IIA :

$$(y^n) \quad \leftrightarrow \quad (A_{\mu456789}, A_{\mu6789}, A_{\mu4589}, A_{\mu4567})$$
$$(x_n) \quad \leftrightarrow \quad (A_\mu, A_{\mu45}, A_{\mu67}, A_{\mu89}) \tag{3.2}$$

By performing a T–duality along x^5, x^7, x^9 according to the geometric recipe given in [17], we may find the corresponding weights for the type IIB embedding and read from table 3 of the

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[6] The dressed charges are the physical charges of the interacting theory, which take into account the dressing of the D-brane naked charges provided by the moduli.

same work their R–R interpretation:

$$\text{type IIB}:$$

$$(y^n) \quad \leftrightarrow \quad (A_{\mu 468}, A_{\mu 568}, A_{\mu 478}, A_{\mu 469})$$

$$(x_n) \quad \leftrightarrow \quad (A_{\mu 579}, A_{\mu 479}, A_{\mu 569}, A_{\mu 578}) \tag{3.3}$$

Now let us consider our generating solution and compute the dressed charges on the point of the moduli space ϕ^∞ defined in eq. (2.8). Implementing eq.(3.1), one finds:

$$(y^0, y^1, y^2, y^3) \;=\; (0, -p_1, -p_2, -p_3)$$

$$(x_0, x_1, x_2, x_3) \;=\; \left(-q_0, -\frac{p_1 q_1}{p_1 + p_2}, \frac{p_1 q_1}{p_1 + p_2}, 0\right) \tag{3.4}$$

From the above expressions we may deduce consistent microscopic configurations corresponding to the generating solution with the chosen boundary condition on the scalar fields at infinity ϕ^∞. From the type IIB viewpoint we may think of a system of D3–branes intersecting at non–trivial angles, but in such a way to preserve 1/8 supersymmetry; this can be achieved if the relative rotation between each couple is a $SU(3)$ rotation, [31]. The configuration is depicted in table 1.

	ϕ_1	ϕ_2	ϕ_3
N_0	$\pi/2$	$\pi/2$	$\pi/2$
N_1	$\pi/2$	0	π
N_2	π	$\pi/2$	0
N_3	θ	$\pi - \theta$	$\pi/2$

Table 1: The position of the D3–branes on the compactifying torus; ϕ_i is the angle on the (x^{2i+2}, x^{2i+3}) torus and θ is a generic non–trivial angle. For each couple of constituent D3-branes, it follows that $\sum_{i=1}^3 (\phi_i^{(\alpha)} - \phi_i^{(\beta)}) = 0 \bmod 2\pi \;\forall \alpha, \beta = 1, ..., 4$, this ensuring that it is a configuration of 4 (bunches of) D3–branes at $SU(3)$ angle, [31, 32]. Notice that the above configuration has been chosen in such a way that setting $\theta = 0$ one recovers a four parameters solution, namely 4 bunches of D3–branes orthogonally intersecting.

Using eq.s (3.3) the first three sets of branes (N_0, N_1, N_2) may be associated with the charges x_0, y^1, y^2 respectively, i.e. with charge along the 3–cycles (579), (568), (478), while the fourth set, N_3, with y^3, i.e. the charge along (469). In fact, due to the non–trivial angle θ, the fourth set of N_3 branes induces D3–brane charge on the cycles (579) (contributing to x_0), (479) (represented by x_1) and (569) (represented by x_2).

As far as the type IIA microscopic interpretation is concerned, we may consider the configuration of D0 and D4–branes obtained by T–dualizing the type IIB one described above along the directions x^5, x^7, x^9. The corresponding system may be deduced from eq.s (3.2) and (3.4) and consists of a set of coinciding D0–branes with electric charge $-x_0$ (the minus sign is required by consistency with the construction in [17] and will be discussed in the sequel) and three sets of coinciding D4–branes along the four–cycles (6789), (4589) and (4567) with magnetic charges y^1, y^2, y^3. In addition there is a magnetic flux (related to the angle θ in the T–dual type IIB configuration, [33, 34]) switched on the world volume of the latter brane (i.e. along (4567)). This flux induces an effective D0 charge (contributing to x_0) and effective D2 charges along the two–cycles (45) and (67) (represented by x_1 and x_2, respectively). The presence of this flux is also consistent with the fact that the axions in the type IIA embedding are interpreted as coming from the B_{MN} tensor in ten dimensions. Indeed, let us briefly recall the general argument relating the presence of a flux on one D4–brane with an effective D2–brane charge (electric in our framework) and a non–trivial B_{ij} background field. As well known, B field components enter non–trivially in the Dp–brane action via the WZ term:

$$\mu_p \int_{W_{p+1}} \left(C \wedge e^{\mathcal{F}} \right)_{p+1} \tag{3.5}$$

where $\mathcal{F}$ is the gauge invariant combination $\mathcal{F} = 2\pi\alpha' F + \hat{B}$ ($\hat{B}$ being the pull–back of the B field). Hence, from the supergravity point of view, one would indeed expect new charges representing extra D$(p-2)$ effective charges at the microscopic level as well as non–trivial bulk B field components in the solution (see for instance [35]). In fact, this is precisely what we get. As shown for instance in [32, 36], for a suitable choice of the flux (which, albeit giving smaller brane charges via world–volume Chern–Simons coupling, modifies the supersymmetry projections imposed by the D–brane background) the above configuration can preserve 1/8 of the original supersymmetry. Consider the D4–brane configuration described above in the general situation in which the magnetic fluxes are non vanishing on all the three planes (45), (67), (89) (i.e. $\mathcal{F}_{45}$, $\mathcal{F}_{67}$, $\mathcal{F}_{89} \neq 0$). From eq. (3.5) we may deduce the effective (electric) D2 and D0–brane charges. For instance, the effective D2–brane charge along (45) (which is the electric dual object of the D4–brane wrapped on (6789)) is:

$$\# \text{ of } D2 \text{ brane (along cycle 45)} = \frac{1}{2\pi} \left(\int_{T_{67}} Tr\, \mathcal{F}_{67} + \int_{T_{89}} Tr\, \mathcal{F}_{89} \right) \tag{3.6}$$

which in our conventions is represented by the electric dressed charge x_1. Similarly, we may compute the effective $D2$ charges along the other two–cycles. Notice that the only non–vanishing components of $\mathcal{F}$ can only be $\mathcal{F}_{45}$, $\mathcal{F}_{67}$, $\mathcal{F}_{89}$ (a $\mathcal{F}_{56} \neq 0$ component would imply, for instance,

a new 4 dimensional magnetic effective charge out of the four at disposal in the central charge normal gauge), and the three axions come precisely from those components of the B field.

On our particular solution the D2–brane charge along (89), i.e. x_3, is zero, while (45) and (67) charges are opposite one to each other ($x_1 = -x_2$), see (3.4). In fact, considering eq.(3.6) written also for the other 2–cycles, one can easily see that switching on a magnetic flux, as we do, only on the D4–branes lying along (4567) and not on the other two bunches of D4–branes, is consistent with having no D2–brane charge along (89), i.e. $x_3 = 0$, and opposite D2–brane charge along (45) and (67), i.e. $x_1 = -x_2$. These charges turn out to be proportional to the *fifth parameter* q_1: sending q_1 to zero the fluxes vanish together with the B fields (axions) and we recover the four parameter solution discussed in [17].

Let us now come to our final goal that is to make the macroscopic/microscopic correspondence precise, namely to give the precise matching between the parameters characterizing the microscopic and the macroscopic configurations, respectively:

$$N_0 \, , \, N_1 \, , \, N_2 \, , \, N_3 \, , \, \theta \; \longleftrightarrow \; q_0 \, , \, q_1 \, , \, p_1 \, , \, p_2 \, , \, p_3$$

The type IIA and IIB D–brane configurations discussed above as the microscopic counterparts of our generating solution, were suggested in [16] as candidates for the microscopic representation of the 5–parameter solution, whose macroscopic description was then missing. Let us focus for the moment on the type IIA embedding. In order to make contact with this literature, let us use an equivalent representation of the dressed charges, related to the central charges by an $SO(8)$[7] transformation:

$$z_{ij} = x_{ij} + iy^{ij} = -\frac{1}{\sqrt{2}} \left(\Gamma^{AB} \right)_{ij} Z_{AB} \tag{3.7}$$

where the couple (ij) indicizes the two times antisymmetric representation of $SO(8)$. The real and imaginary parts of z_{ij} are the $N = 8$ electric and magnetic dressed charges in the basis of weights of the **56** of $E_{7(7)}$ defined in [17] and listed in table 3 of the same paper.

When Z_{AB} is skew–diagonal the matrix $\Gamma \equiv \left(\Gamma^{AB} \right)_{ij}$ has the form:

$$\Gamma = \begin{pmatrix} 1 & -1 & -1 & 1 \\ -1 & 1 & -1 & 1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & -1 & -1 \end{pmatrix} \tag{3.8}$$

the electric (x) and magnetic (y) charges are non vanishing only for (ij) equal to (12),(34),(56)

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[7]The subgroup of $SU(8)$ which does not "mix" electric and magnetic charges.

and (78). From eq.s (2.11) and (3.7) we may read off the values of the charges x_{ij} and y^{ij}:

$$\begin{aligned}
x_{78} + iy^{78} &\equiv \frac{1}{\sqrt{2}}(Z_1 + Z_2 + Z_3 + Z_4) = 0 + iq_0 \\
x_{12} + iy^{12} &\equiv -\frac{1}{\sqrt{2}}(Z_1 - Z_2 - Z_3 + Z_4) = -\frac{q_1 p_1}{p_1 + p_2} - ip_1 \\
x_{34} + iy^{34} &\equiv -\frac{1}{\sqrt{2}}(-Z_1 + Z_2 - Z_3 + Z_4) = \frac{q_1 p_1}{p_1 + p_2} - ip_2 \\
x_{56} + iy^{56} &\equiv -\frac{1}{\sqrt{2}}(-Z_1 - Z_2 + Z_3 + Z_4) = 0 - ip_3
\end{aligned} \tag{3.9}$$

The relation between this representation of the dressed charges and the one in eq.(3.4), which is related to the choice of the STU model in which the generating solution has been worked out, is the following:

$$\begin{aligned}
\{y^{12}, y^{34}, y^{56}, y^{78}\} &= \{y^1, y^2, y^3, -x_0\} \\
\{x_{12}, x_{34}, x_{56}, x_{78}\} &= \{x_1, x_2, x_3, y^0\}
\end{aligned} \tag{3.10}$$

where symplectic transformation $x_0 = -y^{78}$, $y^0 = x_{78}$, as discussed in [17], is related to the feature of our STU model (both in type IIA and type IIB cases) of being embedded non–perturbatively in the larger $N = 8$ theory and therefore is required in order for the truncation to be consistent[8]. The actual D0–brane effective charge is thus $y^{78} = q_0$.

The precise correspondence between the dressed charges (in the two representations) and the parameters associated with the microscopic configurations previously discussed (that is those characterizing the type IIB configuration of table 1: $N_0, N_1, N_2, N_3, \theta$) is represented in table 2.

Finally, according to relations (3.4) or (3.9) and table 2 we finally get the precise macroscopic/microscopic correspondence[9]:

$$N_0 = q_0 - \frac{(q_1 p_1)^2}{p_3(p_1 + p_2)^2} \ , \ N_1 = p_1 \ , \ N_2 = p_2 \ , \ N_3 \cos^2\theta = p_3 \ , \ \tan\theta = -\frac{q_1 p_1}{p_3(p_1 + p_2)} \tag{3.11}$$

Through equations (3.11) all the microscopic parameters, namely N_0, N_1, N_2, N_3 and the angle θ, are expressed in terms of the quantized charges (p_Λ, q_Λ) characterizing the macroscopic

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[8]Indeed the dimensionally reduced R–R vector A_μ in the STU model is an electric potential, while it is magnetic in the $N = 8$ from the type IIA viewpoint, see table 3 of [17].

[9]Our normalizations are the following. In general the 4 dimensional charge of a wrapped Dp–brane is $Q_p = \hat{\mu}_p \cdot V_p / \sqrt{V_6}$ where $\hat{\mu}_p = \sqrt{2\pi}(2\pi\sqrt{\alpha'})^{3-p}$ is the normalized Dp–brane charge density in ten dimensions. Provided the asymptotic values of the dilatons $b_i(r)$, which parameterize the radii of the compactifying torus and which has been taken to be unitary (see previous section), it turns out that, in units where $\alpha' = 1$, the four dimensional fundamental quanta of charge for *any* kind of (wrapped) Dp–brane is equal to $\sqrt{2\pi}$ and our quantized charges (p_Λ, q_Λ) have been taken in units of that quanta, i.e. they are integer valued.

type IIB D–branes	Charge	type IIA D–branes		
3–brane(468)	0	6–brane	y^0	x_{78}
3–brane(568)	$-N_1$	4–brane(6789)	y^1	y^{12}
3–brane(478)	$-N_2$	4–brane(4589)	y^2	y^{34}
3–brane(469)	$-N_3 \cos^2 \theta$	4–brane(4567)	y^3	y^{56}
3–brane(579)	$N_0 + \sin^2 \theta \, N_3$	0–brane	$-x_0$	y_{78}
3–brane(479)	$\sin \theta \cos \theta \, N_3$	2–brane(45)	x_1	x_{12}
3–brane(569)	$-\sin \theta \cos \theta \, N_3$	2–brane(67)	x_2	x_{34}
3–brane(578)	0	2–brane(89)	x_3	x_{56}

Table 2: The correspondence between type IIB and type IIA charges on the different cycles of the compactifying torus. Notice how a $\theta \neq 0$ contribution induces D2–brane and D0–brane effective charges while for $\theta = 0$ one gets a four parameter configuration.

solution and this finally allows us to characterize "quantitatively" its microscopic structure. An alternative microscopic system which could reproduce our generating solution can be given from the M–theory point of view and consists of three M5–branes with magnetic flux on their world volumes and intersecting on a (compact) line along which N_0 units of momentum has been put.

With the above definitions, the expression of the $E_{7(7)}$ quartic invariant J_4 in the (y^{ij}, x_{ij}) basis,[37], is:

$$J_4 = -4 \left(x_{78} x_{12} x_{34} x_{56} + y^{78} y^{12} y^{34} y^{56} \right) - \left(x_{78} y^{78} + x_{12} y^{12} + x_{34} y^{34} + x_{56} y^{56} \right)^2$$
$$+ 4 \left(x_{78} y^{78} x_{12} y^{12} + x_{78} y^{78} x_{34} y^{34} + x_{78} y^{78} x_{56} y^{56} + x_{12} y^{12} x_{34} y^{34} + x_{12} y^{12} x_{56} y^{56} + x_{34} y^{34} x_{56} y^{56} \right)$$

(3.12)

and consequently, upon use of table 2, one can easily work out the expression of the entropy $S = \pi \sqrt{J_4}$ written in terms of the microscopic parameters:

$$S_{micro} = 2 \, \pi \, \sqrt{\cos^2 \theta \left[N_0 N_1 N_2 N_3 - \frac{1}{4} \sin^2 \theta \, N_3^2 \left(N_1 - N_2 \right)^2 \right]}$$

(3.13)

A derivation of the above formula via microscopic counting techniques should be performed extending the analysis of [5, 11] to tori (also the results of [38] could possibly shed some light in this direction). However, we do not try to perform it here. Let us just notice that for $\theta = 0$ one recovers the usual entropy of the four parameters solution whose derivation via microscopic counting has been carried out, for instance, in [36, 39, 40].

4 Discussion

In the present paper we have worked out the generating solution of four dimensional $N = 8$ BPS black holes in a form which could be easily described, applying the results of [17], in terms of pure D–brane configurations upon toroidal compactification of string (or M) theory. As a result we were able to "pinpoint" the precise correspondence between the microscopic parameters characterizing one of these configurations and the supergravity parameters entering the macroscopic description of the solution.

The relevance of this achievement relies on the possibility on one hand to reconstruct the whole 56–parameter U–duality orbit of $N = 8$ BPS black holes, by acting on our solution by means of $E_{7(7)}$ transformations, and on the other hand to study in a precise fashion the action of dualities on their corresponding microscopic realizations. Starting from the type IIA configuration described in the previous section and performing a T–duality transformation on the whole T^6, one ends up, for instance, with a configuration made of N_0 D6–branes, 3 bunches of (N_1, N_2, N_3) D2–branes along the planes (45),(67),(89) plus effective D4–brane charge. But, more generally, we may also unravel the microscopic properties of pure NS–NS black hole solutions in the same orbit, starting from the corresponding embedding of the STU model defined in [17], or even of mixed NS–NS/R–R solutions. The important point is that having now *both* a macroscopic and a microscopic description of the generating solution one can follow its trasformation throughout the full U–duality orbit.

In this respect a challenging problem is to recover the expression of eq.(3.13) from a microscopic entropy counting point of view, performed on the corresponding D–brane configuration. Knowing how to act on it by means of U–duality can help to shed some light on the actual microscopic degrees of freedom of general BPS black holes, since the generating solution encodes, by definition, all of them. This project is left for future work.

Appendix A

The general expression of the $N = 8$ central charge eigenvalues Z_α in terms of the scalar fields and the charges characterizing the STU model can be worked out making explicit the first order equations (2.3) taking into account relations (2.4). Following [19] we have the following:

$$
\begin{aligned}
ReZ_1 &= \frac{-1}{2\sqrt{-2\,b_1 b_2 b_3}}\,(b_1 q_1 - b_2 q_2 - b_3 q_3 + (a_2\,a_3\,b_1 - a_1\,a_3\,b_2 - a_1\,a_2\,b_3 - b_1\,b_2\,b_3)\;p_0 + \\
&\quad + (a_3\,b_2 + a_2\,b_3)\;p_1 + (a_1\,b_3 - a_3\,b_1)\;p_2 + (a_1\,b_2 - a_2\,b_1)\;p_3) \\
ImZ_1 &= \frac{1}{2\sqrt{-2\,b_1 b_2 b_3}}\,(a_1 q_1 + a_2 q_2 + a_3 q_3 + (a_1\,a_2\,a_3 + a_3\,b_1\,b_2 + a_2\,b_1\,b_3 - a_1\,b_2\,b_3)\;p_0 + \\
&\quad - (a_2\,a_3 - b_2\,b_3)\;p_1 - (a_1\,a_3 + b_1\,b_3)\;p_2 - (a_1\,a_2 + b_1\,b_2)\;p_3 + q_0) \\
ReZ_2 &= (1,2,3) \to (2,1,3) \\
ImZ_2 &= (1,2,3) \to (2,1,3) \\
ReZ_3 &= (1,2,3) \to (3,2,1) \\
ImZ_3 &= (1,2,3) \to (3,2,1) \\
ReZ_4 &= \frac{-1}{2\sqrt{-2\,b_1 b_2 b_3}}\,(b_1 q_1 + b_2 q_2 + b_3 q_3 + (a_2\,a_3\,b_1 + a_1\,a_3\,b_2 + a_1\,a_2\,b_3 - b_1\,b_2\,b_3)\;p_0 + \\
&\quad - (a_3\,b_2 + a_2\,b_3)\;p_1 - (a_3\,b_1 + a_1\,b_3)\;p_2 - (a_2\,b_1 + a_1\,b_2)\;p_3) \\
ImZ_4 &= \frac{1}{2\sqrt{-2\,b_1 b_2 b_3}}\,(a_1 q_1 + a_2 q_2 + a_3 q_3 + (a_1\,a_2\,a_3 - a_3\,b_1\,b_2 - a_2\,b_1\,b_3 - a_1\,b_2\,b_3)\;p_0 + \\
&\quad - (a_2\,a_3 - b_2\,b_3)\;p_1 - (a_1\,a_3 - b_1\,b_3)\;p_2 - (a_1\,a_2 - b_1\,b_2)\;p_3 + q_0) \tag{A.1}
\end{aligned}
$$

where it is meant that all axions and dilatons are r–dependent, i.e. $a_i = a_i(r)$ and $b_i = b_i(r)$.

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